Basic RLC Filter Workshop

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Summary

In this workshop learn the basic principles of electronic filters that use only the passive components of resistors, capacitors, and inductors.

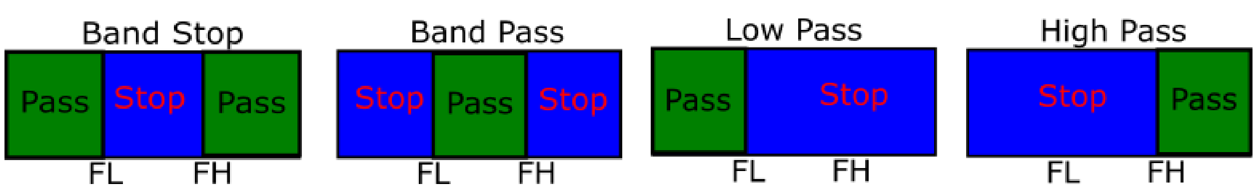
Introduction

Filters are used extensively in electronic circuits to limit the frequency ranges that pass through. For example, if a sensor has a lot of noise on top of a slowly changing valid reading, a low pass filter will greatly reduce the noise but let the signal reading pass through. Radio circuits have filters to only tune into one frequency. Musical instruments have filters for each note.

Passive electronic filters are created from resistors, inductors, and capacitors (RLC). Active filters use transistors and op amps to apply gain or prevent the amplitude loss that occurs in passive filters. An RLC filter is essentially a resistor divider where one part changes resistance (called reactance) based on frequency of the signal passing through. Based on the changing resistance only certain bands of frequencies pass through the filter.

Filter Types

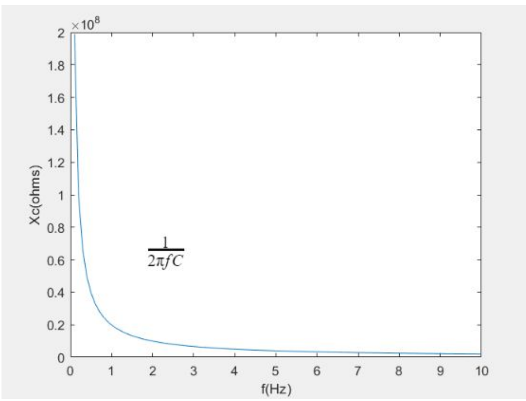
Filters are four basic types: low pass, high pass, bandpass, and band stop. The filter name comes from what part (band) of frequencies pass through.



Filter design is a deep subject using advanced mathematics. In this workshop we only cover passive RLC filters.

Capacitor Filters

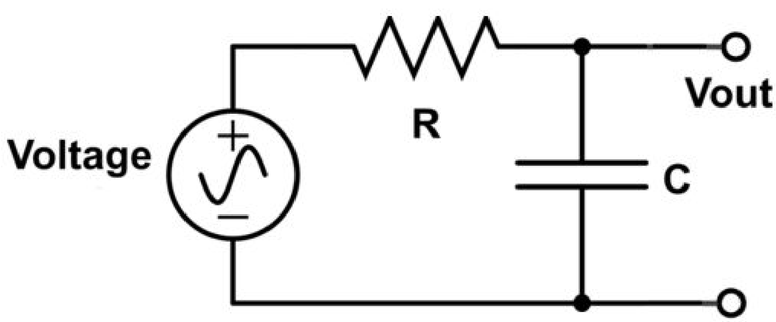
Capacitors store charge but their resistance to current flow varies with frequency. The resistance to AC current is called "Reactance" or X_C . The formula for capacitive reactance is $Xc = \frac{1}{2\pi f C}$ with (C)apacitance in farads and (f)requency in hertz. The reactance decreases with frequency.

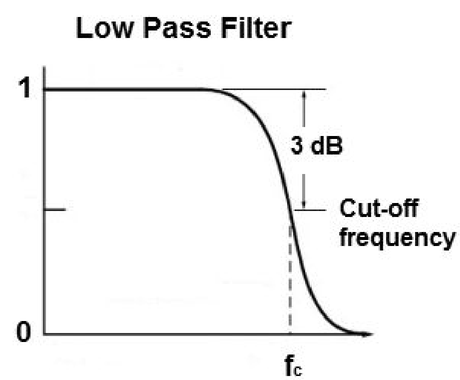


Since the resistance to current decreases the amount of current increases with frequency.

Since capacitors have infinite resistance at 0 hz they are frequently used to block DC voltage between to different parts of the circuit, such as if one part is 5 volts and one part is 3.3 volts.

A resistor-capacitor (RC) low pass circuit is

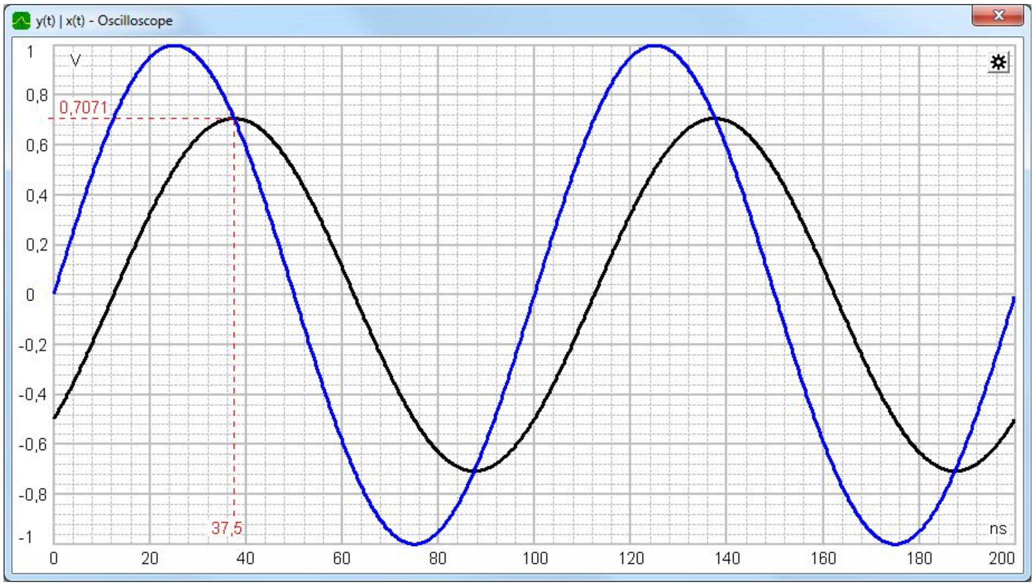




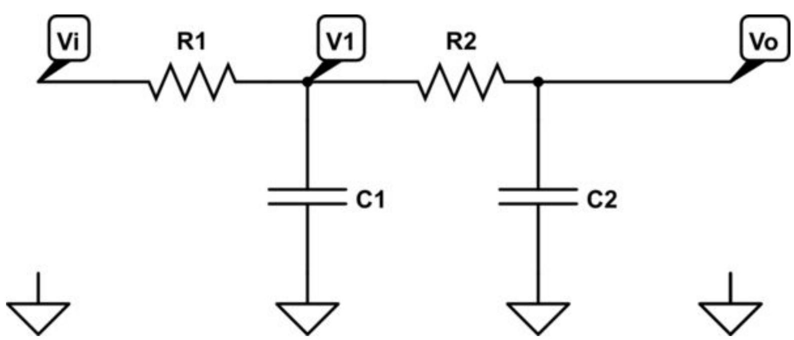
where the cut-off frequency is $f_c = \frac{1}{2\pi RC}$. RC filters shift the phase of the AC signal which can matter if combining several AC signals or if making a phase shift oscillator.

The formula is for the amount of phase shift is:

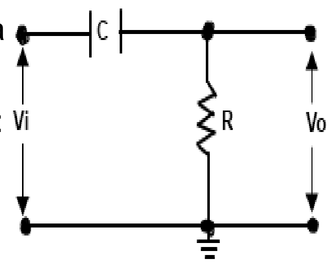
$\theta = tan^{-1} \frac{X_c}{R}$

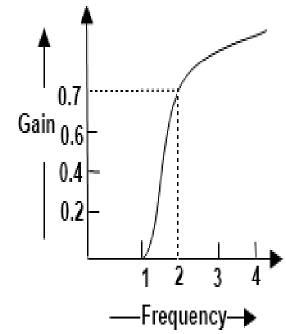


For a steeper cut-off for the filter, stages can be combined or cascaded. Each stage makes the slope steeper of the filter graph of amplitude versus frequency.



An RC high pass filter is similar to a pass, but the resistor and capacitor positions are switched in the circuit:





low

The f_c cutoff is the same formula as before: $f_c = \frac{1}{2\pi RC}$

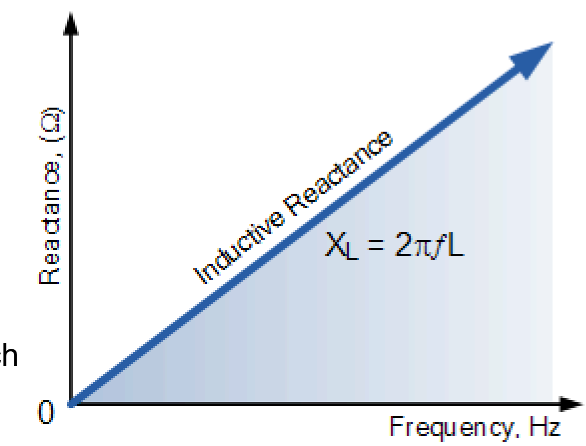
Inductor Filters

An inductor has the opposite behavior to capacitors. Their resistance decreases with frequency.

The formula for inductive reactance is:

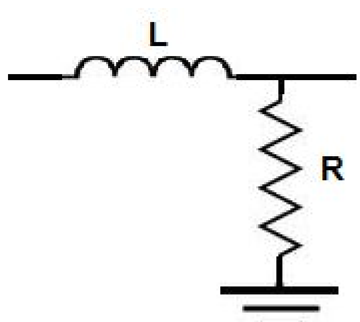
$X_L = 2\pi f L$
with L in henries, (f)requency in hz.

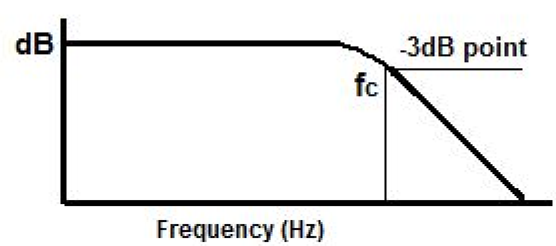
An ideal inductor has no resistance to DC current (which is 0 hz), but steadily increasing resistance to higher frequency AC.



A RL low pass filter circuit is:

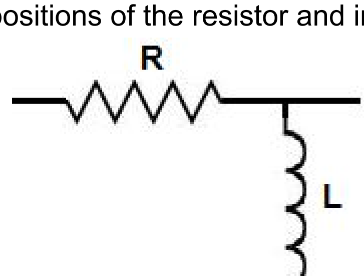
where the cutoff frequency $f_c = \frac{R}{2\pi L}$

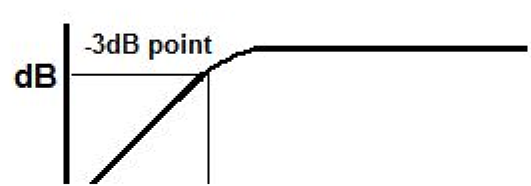




is

Swap the positions of the resistor and inductor in the circuit and we have a high pass filter:





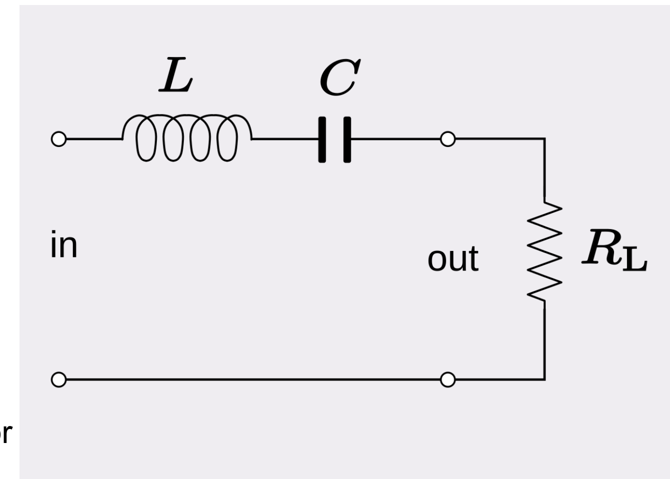
The cutoff frequency (-3dB) is the same formula: $f_c = \frac{R}{2\pi L}$
RL filters can be cascaded for steeper slopes for the frequency range that is blocked.

Inductor and Capacitor Filters

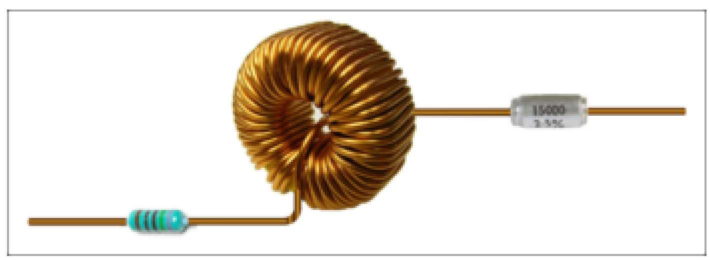
When an inductor and capacitor is combined in series a bandpass filter can be created. The inductor blocks the highest frequencies and the capacitor blocks the lowest frequencies.

The output is taken across the resistor.

Radios with analog tuners use RLC circuits to select or tune in certain frequencies. They use a variable capacitor to change the bandpass frequency to the desired value.



The physical circuit looks quite simple.



The resonant frequency (center frequency) is given by $f_0 = \frac{1}{2\pi\sqrt{LC}}$ with L in Henries and C in farads.

The differential equation for the sum of voltages of a series or parallel RLC circuit is where I(t) is the time varying current.

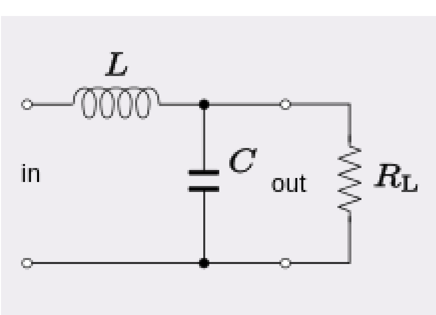
$$\frac{d^2}{dt^2}I(t) + \frac{R}{L}\frac{d}{dt}I(t) + \frac{1}{LC}I(t) = 0\,.$$

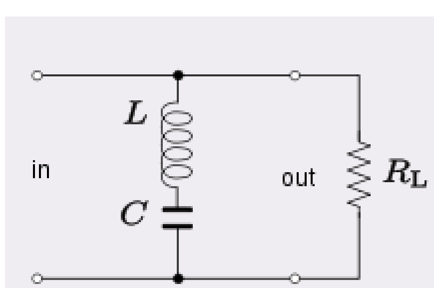
Low pass RLC is

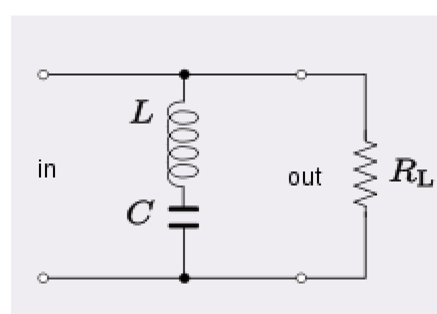
High pass RLC

Series band stop

filter



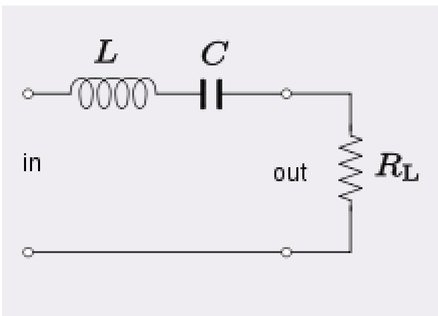


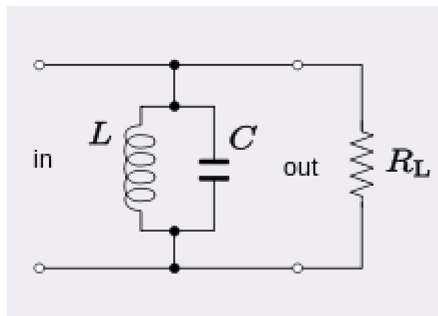


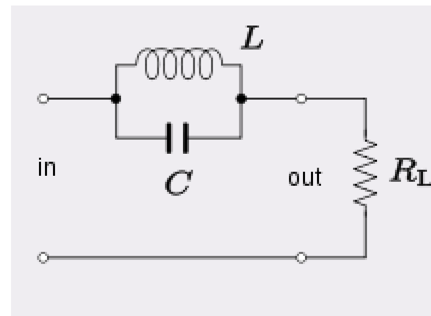
Series RLC bandpass

Parallel band stop

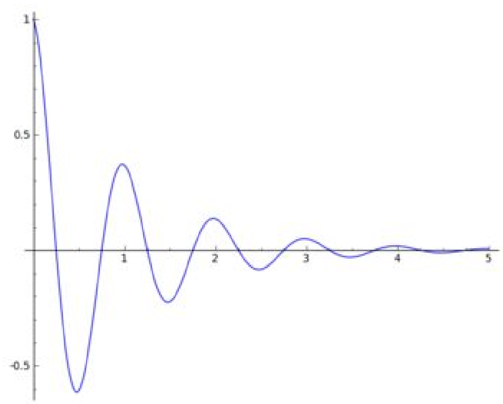
Parallel RLC bandpass







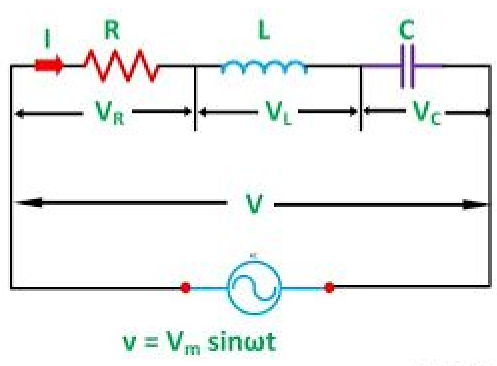
An RLC circuit will oscillate at it's natural resonant frequency, but the signal dampens out exponentially.



With an amplifier such as a transistor included, the bandpass RLC can function as an oscillator.

The phase shift of an RLC circuit is found combining the impedances of the three components. This is not a simple addition of impedances because the phase angle brings in a second dimension.

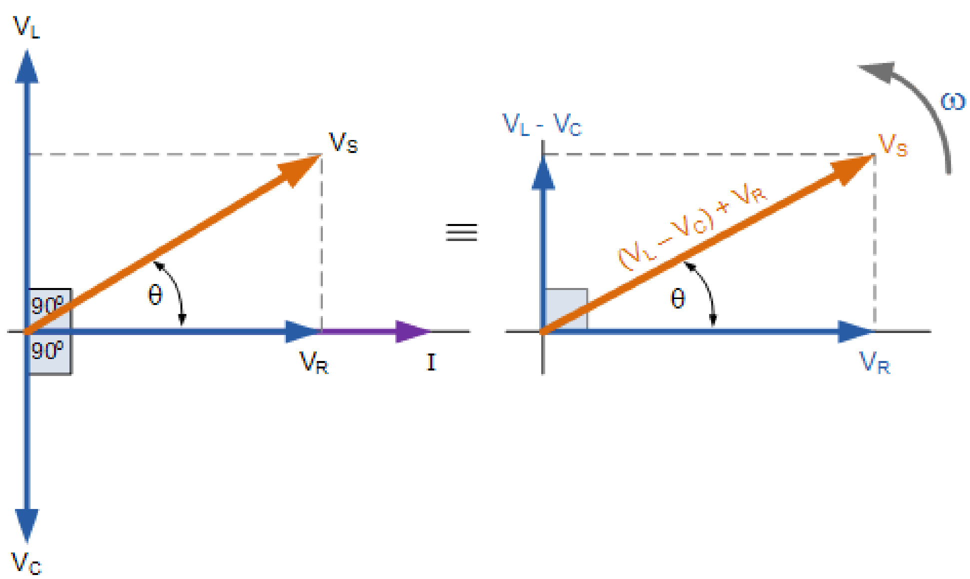
The voltage across the resistor V_R is in phase,
the voltage across the capacitor V_C is 90 degrees



in the negative direction, and the V_L is 90 degrees in the positive direction.

A resistor does not shift the phase of the signal, an inductor shifts it a positive angle, and a capacitor shifts the phase in a negative direction. To find the combined phase shift requires adding the vectors using the Pythagorean theorem for a geometric solution or vector component addition.

In this example the voltages are added. The voltage drop across the capacitor is smaller than the voltage drop across the inductor or resistor. The resulting combination voltage of the signal has a positive phase change.



The combined voltage V or V_S has a magnitude and a phase angle θ found with trigonometry.

Exercises

Preliminaries:

Set up a white noise generator. This can be an arduino running a pseudo-random number generator. It can be an MP3 file downloaded from the Internet and played using a smartphone or laptop and headphone output. Run the signal into the filter, then through an audio amplifier out to a speaker. Look at the output on a spectrum analyser on a smartphone or laptop.

(Audio amplifier chip is LM386.)

(Copy the code from WhiteNoise.ino)

(Download white noise mp3 and play it through the headphone output into the filter circuit.)

Alternate Preliminaries:

Set up a function generator with variable sine wave output. Configure an oscilloscope with settings to match the sine wave frequency and amplitude. Put the filter circuit between the function generator and the oscilloscope. As the sine wave frequency is changed, examine how the amplitude changes on the oscilloscope.

Exercise 1:

Make an RC high pass. Look at the white noise after the filter. Calculate the cutoff frequency. Examine where the cutoff is on the spectrum analyser output.

Exercise 2:

Make an RC low pass using the same components as the low pass filter. Examine the spectrum analyser output.

Exercise 3:

Make a RL low pass. Examine the spectrum analyser output.